UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

FURTHER MATHEMATICS **9231/02**

Paper 2 **October/November 2007**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

6203039236

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\text{m}\,\text{s}^{-2}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages and **3** blank pages.

UNIVERSITY of CAMBRIDGE International Examinations

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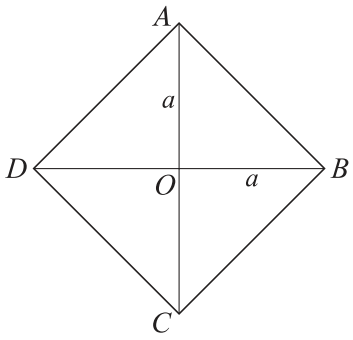
1



The diagram shows a uniform square lamina $ABCD$, of mass $4m$ and with diagonals of length $2a$. The diagonals intersect at O. Find the moment of inertia of the lamina about an axis through O perpendicular to its plane. [1]

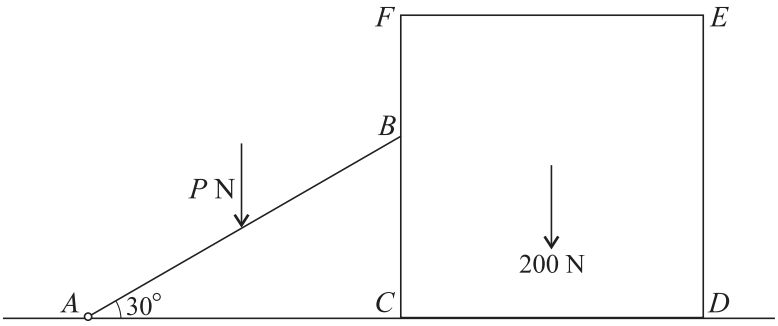
Hence show that the moment of inertia of the triangle AOB about OB is $\frac{1}{6}ma^2$. [2]

After removal from the square, the triangular lamina AOB is free to rotate, without resistance, about a horizontal axis along OB. This lamina is held in a horizontal position and released from rest. Given that the centre of mass of this lamina is at a distance $\frac{1}{3}a$ from OB, calculate the initial acceleration of A. [3]

2 A particle P oscillates in simple harmonic motion between the points A and B, where $AB = 5$ m. When P is 2 m from B its speed is $4\,\text{m}\,\text{s}^{-1}$. Find the number of complete oscillations that P makes in one minute. [5]

The point C is between A and B, and $BC = 3$ m. Find the time taken for P to move directly from B to C. [3]

3



A uniform solid cube, of edge 1 m and weight 200 N, rests on a rough horizontal plane. A light rod AB of length 1.2 m is smoothly hinged at A to a point of the plane and B is in contact with a smooth face of the cube. The rod lies in a vertical plane perpendicular to this face and passing through the centre of mass of the cube. The rod makes an angle of 30° with the horizontal (see diagram). A gradually increasing force of magnitude P N is applied vertically downwards at the mid-point of AB until the cube is on the point of turning about the edge through D. By considering the forces acting on the cube at this instant, find the normal contact force at B and hence show that $P = \dfrac{1000\sqrt{3}}{9}$. [6]

Find the set of values of the coefficient of friction between the cube and the plane for equilibrium to be broken in this way. [3]

4 A smooth hemisphere of radius a has its circular face fixed to the surface of a horizontal table. The centre of this face is O. A particle P of mass m is placed at the highest point of the hemisphere and is given a horizontal velocity u. The particle does not immediately lose contact with the surface of the hemisphere. Assuming all resistances to motion are neglected, find the set of possible values of u. [4]

Show that P loses contact with the surface before OP becomes horizontal. [5]

5 Two smooth spheres A and B, with equal radii and masses m and km respectively, are moving in a straight line on a horizontal plane. They are moving in the same direction with velocity u. Sphere B collides with a vertical barrier and rebounds to collide directly with A. The coefficients of restitution between A and B and between B and the barrier are both $\frac{1}{2}$. Find the velocities of A and B after they collide and hence find the set of values of k such that A and B both reverse direction as a result of the collision. [9]

Find the magnitude of the impulse that acts on A in its collision with B. [2]

6 The blood pressures of 50 randomly chosen patients were measured using an aneroid device and were also measured using an electronic device. The values, x_A using the aneroid device and x_E using the electronic device, are summarised by

$$\Sigma(x_A - x_E) = 12.1, \quad \Sigma(x_A - x_E)^2 = 235.82.$$

The population means for the two devices are denoted by μ_A and μ_E respectively. Find a 90% confidence interval for $\mu_A - \mu_E$. [6]

Interpret 'a 90% confidence interval' in the context of the question. [1]

State, giving a reason, whether it is necessary to assume that the differences in values have a normal distribution. [1]

7 The continuous random variable X is believed to have the distribution function given by

$$\mathrm{F}(x) = \begin{cases} 0 & x < 1, \\ 1 - \dfrac{1}{x^2} & x \geqslant 1. \end{cases}$$

A random sample of 80 observations of X was taken and these values are summarised in the following grouped frequency table. Also shown in the table are the expected frequencies for a χ^2 goodness of fit test.

Values	$1 \leqslant x < 2$	$2 \leqslant x < 3$	$3 \leqslant x < 4$	$x \geqslant 4$
Observed frequency	69	8	2	1
Expected frequency	60	$11\frac{1}{9}$	$3\frac{8}{9}$	5

Show how the value of $3\frac{8}{9}$ is obtained using the distribution function. [2]

Carry out the goodness of fit test at the 10% significance level. [6]

8 In a project, Anna was required to investigate how the mass of a banana was related to its length. She selected a random sample of 6 bananas, from a batch, and carefully measured the length, x cm, and the mass, y grams, of each banana. The summary values were as follows.

$$\Sigma x = 88.5,\ \Sigma x^2 = 1320.29,\ \Sigma y = 593,\ \Sigma y^2 = 62\,777,\ \Sigma xy = 8982.5.$$

(i) Find the product moment correlation coefficient for the sample. [3]

(ii) Use the equation of a suitable regression line to estimate the length of a banana of mass 100 grams from the batch. [5]

(iii) Comment on the reliability of your answer. [1]

9 Log-ons to a certain computer website occur randomly at a uniform average rate of 2.4 per minute. State the distribution of the number N of log-ons that occur during a period of t minutes. [1]

Obtain the probability that at least one log-on occurs during a period of t minutes. [2]

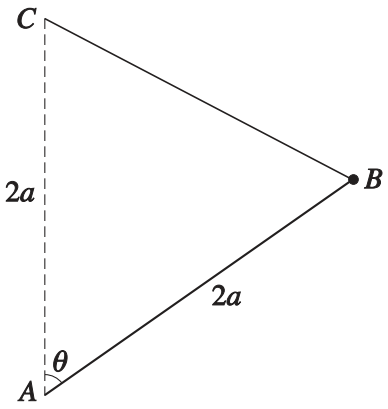
Hence obtain the probability density function of T, where T minutes is the interval between successive log-ons. [3]

Identify the distribution of T and state its mean and variance. [3]

10 At an International Festival 12 comedians had their Intelligence Quotients (IQs) measured. The IQs, x, of the comedians are summarised by $\Sigma(x - 128) = 18$ and $\Sigma(x - 128)^2 = 115$. Assuming that these comedians form a random sample of all comedians, and stating any further assumption you need to make, test at the 5% significance level whether the mean IQ of comedians exceeds 128. [9]

11 Answer only **one** of the following two alternatives.

EITHER



A light rod AB of length $2a$ is freely jointed at A to a fixed point and a particle of mass m is attached to B. A light elastic string, of natural length a and modulus of elasticity kmg, has one end attached to B and the other end attached to a point C which is fixed at a distance $2a$ vertically above A (see diagram). The system is in motion in a vertical plane, without the string becoming slack. If air resistance is neglected, show that

$$\ddot{\theta} = \frac{g}{2a}\left(k\cos\tfrac{1}{2}\theta - (2k-1)\sin\theta\right),$$

where angle $CAB = \theta$. [6]

In the equilibrium position, $\theta = \frac{1}{3}\pi$. Find the value of k. [2]

By substituting $\theta = \frac{1}{3}\pi + \varepsilon$, where ε is small, show that the motion is approximately simple harmonic, and give the period. [6]

OR

An experiment was performed to investigate a possible association between a person's hair-colour and eye-colour. The researcher selected a random sample of 60 people and classified their hair and their eyes as either light or dark. 36 people had light hair and 25 had light eyes. A χ^2 test of independence is to be carried out at the 5% significance level. Denoting by n the number of people with both light hair and light eyes, find the set of values of n for which it would be accepted that there is an association between hair-colour and eye-colour. [11]

Another researcher performed the experiment with a random sample of 40 people. In this sample, of the 24 people with light eyes 7 had light hair. Of the people with light hair 3 had dark eyes. Explain why the use of a χ^2 test on this data would be inappropriate. [3]

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